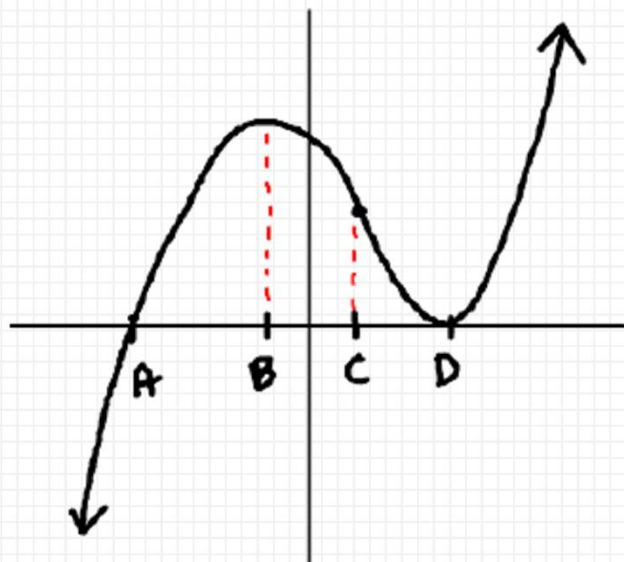


Chapter 4

Section 4.1 – Increasing/Decreasing/Concavity/POI



1. $f(x) = 0$

2. $f(x) > 0$

3. $f(x) < 0$

4. $f'(x) = 0$

5. $f'(x) < 0$

6. $f'(x) > 0$

7. $f''(x) = 0$

8. $f''(x) < 0$

9. $f''(x) > 0$

10. $f'(x) < 0$ and $f''(x) < 0$

11. $f'(x) < 0$ and $f''(x) > 0$

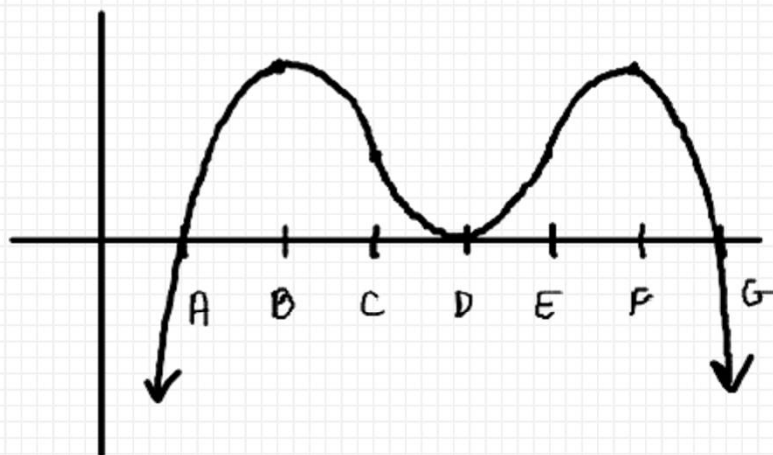
12. $f'(x) > 0$ and $f''(x) < 0$

13. $f'(x) > 0$ and $f''(x) > 0$

14. $f'(x) = 0$ and $f''(x) < 0$

15. $f'(x) = 0$ and $f''(x) > 0$





16. $f(x) = 0$

17. $f(x) > 0$

18. $f(x) < 0$

19. $f'(x) = 0$

20. $f'(x) < 0$

21. $f'(x) > 0$

22. $f''(x) = 0$

23. $f''(x) < 0$

24. $f''(x) > 0$

25. $f'(x) < 0$ and $f''(x) < 0$

26. $f'(x) < 0$ and $f''(x) > 0$

27. $f'(x) > 0$ and $f''(x) < 0$

28. $f'(x) > 0$ and $f''(x) > 0$

29. $f'(x) = 0$ and $f''(x) < 0$

30. $f'(x) = 0$ and $f''(x) > 0$



Increasing/Decreasing:

- ① IF $f'(x) > 0$ on $(a, b) \Rightarrow f$ IS INCREASING on $[a, b]$
- ② IF $f'(x) < 0$ on $(a, b) \Rightarrow f$ IS DECREASING on $[a, b]$
- ③ IF $f'(x) = 0$ on $(a, b) \Rightarrow f$ IS CONSTANT on $[a, b]$

Concavity:

- ① IF $f''(x) > 0$ on $(a, b) \Rightarrow f$ IS CONCAVE UP on (a, b)
- ② IF $f''(x) < 0$ on $(a, b) \Rightarrow f$ IS CONCAVE DOWN on (a, b)

Point of Inflection: ANY VALUE C SUCH THAT $f''(x)$ CHANGES SIGN AT $x = C$.



Critical Point: A VALUE C SUCH THAT 1) $f'(c) = 0$

★ C IS IN THE DOMAIN OF f

OR
2) $f'(c)$ DNE

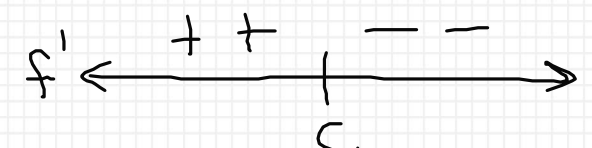
Theorem: IF $f(x)$ HAS AN EXTREME VALUE AT $x = C \Rightarrow$ C IS A CRITICAL PT
LOCAL MIN / LOCAL MAX

CONVERSE: IF C IS A CRIT. PT $\Rightarrow f(x)$ HAS AN EXTREME VALUE AT $x = C$.

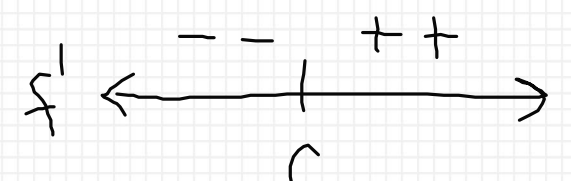
NOT TRUE!!
Ex: $f(x) = x^3$
 $f' = 3x^2 = 0$
 $x = 0$
CRIT. PT
NOT MAX/MIN

First Derivative Test:

① IF c IS A CRIT. PT OF $f(x)$ AND $f'(x)$ CHANGES FROM $+$ TO $-$
AT $x=c \Rightarrow$ REL. MAX @ $x=c$



② IF c IS A CRIT. PT OF $f(x)$ AND $f'(x)$ CHANGES FROM $-$ TO $+$
AT $x=c \Rightarrow$ REL. MIN @ $x=c$.



Second Derivative Test: (VERIFY MAX/MIN)

① IF C IS A CRIT PT OF $f(x)$ AND $f''(c) > 0$ \Rightarrow REL. MIN @ $x=c$
 f IS CC UP AT $x=c$.

② IF C IS A CRIT. PT OF $f(x)$ AND $f''(c) < 0$ \Rightarrow REL MAX @ $x=c$
 f IS CC \downarrow AT $x=c$

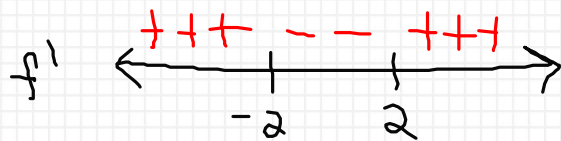


Example: $f(x) = x^3 - 12x - 5$

THIS ESTABLISHES CRIT. PT

$$f' = \begin{cases} 3x^2 - 12 = 0 \\ 3(x^2 - 4) = 0 \\ 3(x+2)(x-2) = 0 \end{cases}$$

CR. PTS: $x = \pm 2$



f INCR $(-\infty, -2] \cup [2, \infty)$

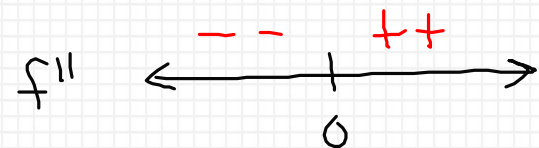
f DECR $[-2, 2]$

SINCE f' CHANGES FROM + TO - AT $x = -2 \Rightarrow$
REL. MAX AT $x = -2$

SINCE f' CHANGES FROM - TO + AT $x = 2 \Rightarrow$
REL. MIN @ $x = 2$.

$$f'' = 6x = 0$$

$$x = 0$$



f CONCAVE DOWN ON $(-\infty, 0)$
SINCE $f'' < 0$

f CONCAVE UP ON $(0, \infty)$
SINCE $f'' > 0$.

POINT OF INFLECTION @ $x = 0$ SINCE
 f'' CHANGES SIGN @ $x = 0$



Example (2nd Derivative Test): $f(x) = x^3 - 12x - 5$

$$\left. \begin{array}{l} f' = 3x^2 - 12 = 0 \\ \vdots \\ x = \pm 2 \end{array} \right\} \begin{array}{l} \text{ESTABLISHES} \\ \text{CRIT PT} \end{array}$$

$$f'' = 6x$$

$$f''(-2) = -12 < 0 \Rightarrow \text{REL. MAX @ } x = -2.$$

$$f''(2) = 12 > 0 \Rightarrow \text{REL. MIN @ } x = 2$$



Example: $f(x) = e^x(x^2 - x - 5)$

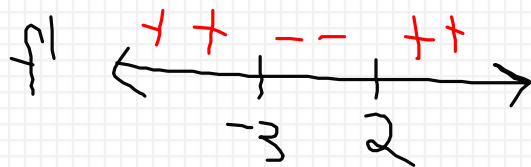
$$f' = e^x(2x-1) + (x^2-x-5)e^x$$

$$0 = e^x(2x-1+x^2-x-5)$$

$$0 = e^x(x^2+x-6)$$

$$0 = e^x(x+3)(x-2)$$

$$x = -3, 2$$



REL MAX @ $x = -3$ SINCE f'
CHANGES FROM + TO - AT $x = -3$

REL. MIN @ $x = 2$ SINCE f'
CHANGES FROM - TO + AT $x = 2$.



Example: $f(x) = x^{\frac{4}{3}} + x^{\frac{1}{3}}$

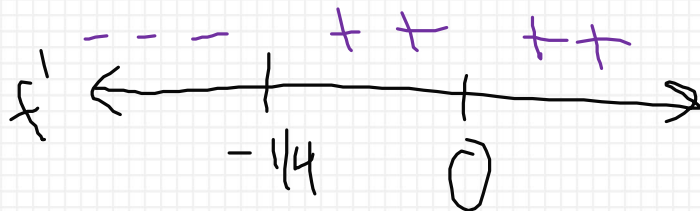
$$f' = \frac{4}{3}x^{\frac{1}{3}} + \frac{1}{3}x^{-\frac{2}{3}} = 0$$

$$0 = \frac{1}{3}x^{\frac{-2}{3}} \left(4x^{\frac{1}{3}} + 1 \right)$$

$\underbrace{\hspace{1.5cm}}_{x^{\frac{2}{3}}}$

$$x = 0, -1/4$$

$f'(0) = \text{DNE}$ $f'(-1/4) = 0$



REL. MIN @ $x = -1/4$ SINCE
 f' CHANGES FROM $-$ TO $+$
 AT $x = -1/4$.



Homework/Classwork:

AP Packet: # 5-23, 26 (skip #14)

